Opening the ‘Black Box’ of Efficiency Measurement: Input Allocation in Multi-Output Settings*

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Abstract:

We develop a new Data Envelopment Analysis (DEA)-based methodology for measuring the efficiency of Decision Making Units (DMUs) characterized by multiple inputs and multiple outputs. The distinguishing feature of our method is that it explicitly includes information about output-specific inputs and joint inputs in the efficiency evaluation. This contributes to opening the ‘black box’ of efficiency measurement in two different ways. First, including information on the input allocation substantially increases the discriminatory power of the efficiency measurement. Second, it allows to decompose the efficiency value of a DMU into output-specific efficiency values which facilitates the identification of the outputs the manager should focus on to remedy the observed inefficiency. We demonstrate the usefulness and managerial implications of our methodology by means of a unique dataset collected from the Activity Based Costing (ABC) system of a large service company with 290 DMUs.

Key words: efficiency measurement, DEA, input allocation, efficiency decomposition, ABC systems

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1. Introduction

Efficiency analysis of production activities is an important issue for practitioners as well as an area of contemporary interest in both the operations research and economics literature (see, for example, Färell et al. (1994), Cooper et al. (2000), Fried et al. (2008), Cook and Seiford (2009) for reviews). The goal of such analysis is to evaluate the efficiency of a DMU (i.e. Decision Making Unit, which is typically a business unit, office or branch of a private or public sector company) by comparing its input-output performance to that of other DMUs operating in a similar technological environment (typically other business units of the same company). Amongst the efficiency measurement techniques, Data Envelopment Analysis (DEA) has become popular both as an analytical research instrument and as a practical decision-support tool. DEA is a production frontier technique with the distinguishing feature that it is nonparametric in nature, which means that it does not resort to some (typically unverifiable) parametric/functional specifications for the production technology but rather "lets the data speak for themselves".

Still, existing DEA methods essentially provide a "black box" treatment of efficient production behavior, because they only use information on inputs and outputs (and sometimes their prices) to evaluate the efficiency of each DMU. What happens inside the "black box", i.e. how inputs and outputs are exactly linked to each other, does not enter the analysis. However, including such information can improve the discriminatory power of efficiency models without needing to resort to unverifiable assumptions. In this study, we develop a DEA-based methodology for efficiency analysis that explicitly includes information about the allocation of inputs to outputs. In the application, we use Activity Based Costing (ABC) data of a large service company with 290 DMUs to show the practical relevance and managerial implications of our newly developed methodology.

The methodology we develop is rooted in the structural efficiency measurement approach initiated by Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983) and Varian (1984).[1] This approach starts from a structural model of efficient production behavior and characterizes inefficiency as deviations from this model. Cherchye et al. (2008) adapted this approach to a multi-output setting that specifically accounts for economies of scope in production. The distinguishing feature of their methodology is that it explicitly recognizes that each different output is characterized by its own production technology, while accounting for interdependencies between the different output-specific technologies. Building on the original idea of Cherchye et al. (2008), we propose an efficiency measurement method that distinguishes between output-specific inputs and joint inputs.[2] The unique feature of our methodology is that we explicitly include information about the

[1] See also Banker and Maindiratta (1988) for an early study on the interrelationship between DEA and this structural approach to analyzing efficient production behavior.

[2] Output-specific inputs are inputs that can be fully allocated to an output. For instance, when the input "labor" is used to produce two products and we can observe that 30% of labor time is used for product 1 and 70% for product 2, then labor can be decomposed into output-specific inputs "labor product 1" and "labor product 2". By

allocation of the output-specific inputs to the outputs. This practice opens the black box of efficiency measurement in two different ways. First, including information on the allocation of output-specific inputs substantially increases the discriminatory power of the efficiency measurement: our efficiency measurement method has more power to identify inefficient production behavior. In turn, this should lead to more actions for efficiency improvement and, consequently, higher realized cost reductions. Second, our methodology allows us to decompose the overall efficiency score of a DMU into output-specific efficiency scores and their respective weights in the DMU's overall efficiency. Such a decomposition is particularly attractive from a practical point of view, as it directly identifies the outputs on which DMU-managers should principally focus to remedy the observed inefficiency. Thus, our methodology should lead to more improvement actions and support managers to focus these improvement actions on the sources that contribute the most to the observed inefficiency.

As we describe in detail in the following sections, the benefits of our methodology hinge on the availability of information about the allocation of inputs to outputs. Although perfect information about the allocation of inputs to outputs is hardly ever available, many large companies - which are typically considered in efficiency analyses - have well developed costing systems that provide information about the allocation of inputs (i.e. cost categories) to outputs (i.e. products) (Cooper and Kaplan 1998). While our methodology does not put any restrictions on the type of costing system that the company uses, we will demonstrate the usefulness of our methodology with data coming from an ABC system (Cooper and Kaplan 1988). ABC systems are widely used in practice for supporting various operational and strategic decisions such as pricing, cost reduction, product development, product mix decisions, and process re-engineering (Cooper and Kaplan 1998, Gosselin 2007). The philosophy underlying ABC is that costs (or inputs) are first allocated to activities (i.e. the first stage of the ABC system) and, subsequently, these activity costs are allocated to the products (or outputs) (i.e. the second stage of the ABC system).[3]

ABC provides a natural complement to our new DEA-based methodology for two reasons. First, proponents of ABC argue that the inclusion of activities in the transformation process from inputs to outputs leads to an accurate reflection of the complex production processes of companies with multiple inputs and outputs (Cooper and Kaplan 1988, 1998). Specifically, ABC systems approximate the underlying production processes, which enables us to obtain accurate information about the decomposition of the inputs to the outputs without having to rely on (unverifiable) assumptions regarding the production technology. Second, ABC data are especially useful for production processes with multiple inputs and multiple outputs. Typically, the inputs of such production processes cannot be attributed to the outputs in a direct way, but the inclusion of activities

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contrast, joint inputs cannot be allocated to specific outputs. A typical example of a joint input is the compensation package of a CEO.

[3] Note that the ABC literature uses “expenses” or “costs” rather than “inputs”. However, expenses, costs and inputs all refer to “resources that are used to produce outputs”. For the sake of clarity and for maintaining consistency with the DEA terminology, we will use “inputs” in this study.

between the inputs and outputs makes it possible to obtain accurate information about the decomposition of the inputs to the outputs. In other words, ABC data can be considered as the operationalization of the input decomposition that is central in our proposed methodology.[4]

Summarizing, our study contributes to both the DEA literature and the accounting literature. The main contribution to the DEA literature is that, although we adopt minimal assumptions regarding the underlying production technology, we are able to set up an efficiency measurement methodology with considerable discriminatory power by explicitly including information about the allocation of inputs to outputs. Furthermore, the explicit inclusion of information about the allocation of the inputs to the outputs enables us to decompose the overall efficiency score in output-specific efficiencies, which can significantly improve managerial decision-making. Importantly, we also show that the dual formulation of our efficiency measurement model has an interpretation similar to standard DEA models. This enables the practitioner to interpret and compare scores easily. More generally, it presents our newly proposed methodology as a natural extension of the existing DEA methodology. In fact, as we will demonstrate in our application, the model allows for any extension that is often added to DEA analyses, such as controlling for exogenous factors or incorporating weight restrictions.

This study also contributes to the accounting literature by showing that ABC information can be fruitfully applied for evaluating productive efficiency of business units, a purpose of ABC systems that has not been identified in prior studies. While some critics of ABC systems argue that the development costs of such systems are too high compared to the benefits that they generate, our application shows that the use of ABC information for evaluating productive efficiency can help companies to significantly reduce their cost level. In other words, the usefulness of ABC systems for efficiency analyses can be a major decision criterion to invest in such systems.

The remainder of the paper is organized as follows. Section 2 introduces our methodology. Section 3 presents our empirical application and discusses the managerial implications. Section 4 concludes and presents some opportunities for future research.

2. Methodology

In Section 2.1 we explain the role of ABC systems in allocating inputs to outputs. This will set the stage for our methodological research question treated in the following sections. Sections 2.2 to 2.4 then provide a formal presentation of our methodology. In Section 2.2 we present a characterization of our efficiency concept for multi-output production, which will provide the theoretical motivation for

[4] Remark that ABC systems do not provide perfect information about the decomposition of the inputs to the outputs (see for instance Datar and Gupta (1994) and Labro and Vanhoucke (2007)). However, ABC systems are considered as the most accurate approximation of the decomposition of inputs to outputs (Bhimani et al. 2007). As a result, including ABC data in efficiency analyses should lead to the most reliable results given that perfect information about the decomposition of inputs to outputs is not available in general or not available at a reasonable cost.

the proposed efficiency assessment methodology. Specifically, we establish the equivalence between, on the one hand, Pareto-Koopmans output efficiency and, on the other hand, multi-output cost efficiency. [5] In Section 2.3 we demonstrate that the cost minimization characterization provides a useful starting point for DEA-type efficiency measurement. Specifically, we present our measure of multi-output cost efficiency, and the associated decomposition in output-specific cost efficiencies. Finally, in Section 2.4 we show that the efficiency measure can be computed through simple linear programming. In addition, we extend this analysis to situations in which no (or only limited) price information is available. Here we also introduce the dual presentation of our efficiency measurement model, which will provide a clear link with existing DEA models.

As a preliminary remark, we note that our following analysis will (only) use the production assumptions of free output disposability and convexity of producible output sets (see below). DEA applications often use additional production assumptions (e.g. related to the nature of the returns-to-scale).[6] To keep our discussion simple, we will abstract from explicitly discussing such additional assumptions in our analysis. However, we emphasize that such assumptions can be easily incorporated into our method, i.e. by including the corresponding (linear) DEA restrictions in the linear programs presented in Section 2.4.

2.1 Input allocation with ABC data

A typical production process transforms multiple inputs into multiple outputs. Inputs can be considered as resource/cost categories and can be expressed in monetary terms. Outputs can be individual products, product categories, customers or market segments. Ideally, the allocation of inputs to outputs is perfectly observable. However, this information is not available and companies use costing systems to allocate inputs to outputs. During the last two decades, production processes have become more complex, which necessitates the development of more refined costing systems.[7] ABC is probably the most widespread costing system. An ABC analysis first allocates inputs to activities by means of resource drivers. In a second stage, the cost of the activities is allocated to the

[5] Pareto-Koopmans efficiency is a frequently used concept in the DEA literature on technical efficiency; see, for example, Charnes et al. (1985) for an early study. Next, cost efficiency (or cost minimizing behavior for a given output) is a well-established concept of economic efficiency. For example, cost minimization is often applicable as the appropriate behavioral assumption for public service companies (with exogenously given outputs). More generally, cost minimization is known to be a necessary condition for profit maximizing behavior (see, for example, Varian (1992) for a general discussion).

[6] See, for example, Cooper et al. (2000) for an overview of production assumptions that are frequently used in DEA applications.

[7] It should be noted that some inputs such as material costs can be directly allocated to the outputs. Costing systems are thus only used for allocating inputs that cannot be allocated in a direct way to the different inputs (i.e. overhead costs). Remark that nowadays more than 80% of the total inputs do not have a clear relationship with the outputs and should thus be allocated to the outputs via the costing system. In service companies, which often use DEA-based methodologies to assess the efficiency of their business units, nearly all costs should be allocated via the costing system.

outputs by means of activity drivers. Figure 1 presents a graphical representation of an ABC costing system.

-Insert Figure 1 about here -

In an ABC system, outputs can be considered as consumers of activities and activities can be considered as consumers of inputs. This implies that outputs can be entirely written in terms of activities and in terms of inputs. Thus, by relying on ABC systems, we know which percentage of an input is used for the production of a certain output. In other words, ABC systems generate accurate information about the input decomposition, which directly relates to the distinguishing feature of our newly developed methodology.

ABC systems also enable us to distinguish between output-specific inputs and joint inputs. While ABC systems provide a way to allocate inputs to outputs, such systems also implicitly recognize that some inputs cannot be allocated to the different outputs in an accurate way. Specifically, ABC systems distinguish between different types of inputs that are necessary to produce the outputs (Cooper and Kaplan 1991). “Unit level” inputs such as direct labor, materials, and machine costs are consumed at the unit level and increase each time a unit of an output is produced. “Batch level” inputs such as setup costs and inspection costs are made to process another batch of products, “product-level” inputs such as costs for product engineering are triggered for every product that is introduced in the product portfolio, and “facility level” inputs such as compensation of the DMU management and costs for maintenance of the buildings are used to support the facility or DMU. It is important to mention that “facility level” inputs lack any relationship with the activities and the outputs (Cooper and Kaplan 1991; 1998). As efficiency assessments can be biased by allocating inputs that have no cause-and-effect relationship with the outputs (i.e. an increase in the number of outputs does not lead to an increase in the inputs), we will consider facility level inputs as joint inputs in our efficiency assessment.[8]

2.2 Characterizing efficient multi-output production behavior

Practical efficiency analysis starts from a data set with T DMUs, which produce M outputs. As indicated above, at the input side, we make the distinction between output-specific inputs and joint inputs. Specifically, we assume N^{spec} output-specific inputs (e.g. which the ABC system can allocate) and N^{join} joint inputs (e.g. which the ABC system cannot allocate). In this and the next section we will

[8] Although proponents of ABC systems recognize that facility level inputs lack any relationship with the activities of the ABC system and thus with the outputs, ABC systems sometimes allocate these inputs to the activities and the outputs. The main reason for allocating such inputs is that ABC information is often used for pricing. Not allocating some inputs will lead to unit costs of the outputs that are too low and prices that do not cover the total amount of costs that have been made to produce the outputs.

assume that the data set also contains the prices of the (output-specific and joint) inputs. We will relax this assumption in Section 2.4. In fact, exact price information will not be available for our empirical application in Section 3.

More formally, we use the following notation for the observed quantities and prices of each DMU t ($1 \leq t \leq T$). First, we observe an M-vector of outputs $\mathbf{y}_t \in \mathbb{R}_+^M$; we use $\mathbf{y}_t = \left(y_t^1, \ldots, y_t^M\right)$ with each entry y_t^m representing the amount that DMU t produces of the m-th output ($1 \leq m \leq M$). Next, we observe an N^{spec}-vector of output-specific inputs $\mathbf{q}_t^m \in \mathbb{R}_+^{Nspec}$ for each individual output m, and an N^{join}-vector of joint inputs $\mathbf{Q}_t \in \mathbb{R}_+^{Njoin}$. Correspondingly, we observe a price vector $\mathbf{p}_t \in \mathbb{R}_+^{Nspec}$ for the output-specific inputs and a price vector $\mathbf{P}_t \in \mathbb{R}_+^{Njoin}$ for the joint inputs. The full data set can be summarized as

$$S = \left\{ \left(\mathbf{y}_t, \mathbf{q}_t^1, \ldots, \mathbf{q}_t^M, \mathbf{Q}_t, \mathbf{p}_t, \mathbf{P}_t \right) \middle| t = 1, \ldots, T \right\}.$$

In what follows, we use $\mathbf{p}_t^{'} \left(\sum_{m=1}^{M} \mathbf{q}_t^m \right) + \mathbf{P}_t^{'} \mathbf{Q}_t = z_t$ where z_t is the budget (or cost) associated with DMU t.

To define our cost efficiency criterion for DMU t, we characterize the production technology by a vector of production functions $f(\mathbf{q}^1, \ldots, \mathbf{q}^M, \mathbf{Q}) = (f^1(\mathbf{q}^1, \mathbf{Q}), \ldots, f^M(\mathbf{q}^M, \mathbf{Q}))$, where each function $f^m(\mathbf{q}^m, \mathbf{Q})$ represents the maximum quantity of output m that can be produced by the output-specific input $\mathbf{q}^m \in \mathbb{R}_+^{N\,spec}$ and the public input $\mathbf{Q}$. Importantly, the use of a separate production function $f^m(\mathbf{q}^m, \mathbf{Q})$ for each m makes clear that our method incorporates that each different output is characterized by its own production technology. At the same time, we account for interdependencies between the different output-specific technologies through the joint inputs $\mathbf{Q}$; as discussed in the introduction, including joint inputs allows for economies of scope in production (see Cherchye et al. 2008). Throughout the study, we assume that the production functions $f^m(\mathbf{q}^m, \mathbf{Q})$ satisfy free output disposability, i.e. less output never requires more input. Free output disposability is a standard assumption in the DEA literature (see, for example, Varian (1984) and Tulkens (1993) for discussion).

For a given technology (characterized by $f(\mathbf{q}_t^1, \ldots, \mathbf{q}_t^M, \mathbf{Q}_t)$), we use the following definition of a producible output set associated with budget z, prices $\mathbf{p}$ for the output-specific inputs and prices $\mathbf{P}$ for the joint inputs:

$$P\left(z, \mathbf{p}, \mathbf{P}\right) = \{\mathbf{y} \middle| \mathbf{y} \leq f\left(\mathbf{q}^1, \ldots, \mathbf{q}^M, \mathbf{Q}\right) \; for \; \mathbf{p}' \left(\sum_{m=1}^{M} \mathbf{q}^m \right) + \mathbf{P}' \mathbf{Q}_s \leq z\}.$$

Thus, this set contains all output vectors $\mathbf{y}$ that can be produced with the given budget z under the prices $\mathbf{p}$ and $\mathbf{P}$. In what follows, we assume that the sets $P\left(z,\mathbf{p},\mathbf{P}\right)$ are convex. This assumption is again widely used in the DEA literature. It implies that the marginal rates of output transformation are everywhere decreasing along the boundary of the feasible production set (see Petersen (1990), Bogetoft (1996) and Cherchye et al. (2008) for a discussion).

We are now in a position to define efficient production behavior. Specifically, we say that DMU t is efficient if the corresponding inputs $\left(\mathbf{q}_t^1,\ldots,\mathbf{q}_t^M,\mathbf{Q}_t\right)$ yield an output combination $\mathbf{y}_t$ that is Pareto-Koopmans output efficient for the producible output set $P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right)$. Pareto-Koopmans output efficiency is a well-established efficiency criterion in the efficiency measurement literature. It has the following formal definition:

Definition 1: *For DMU t the output vector* $\mathbf{y}_t$ *is Pareto-Koopmans output efficient if*

$$\textit{for all } \mathbf{y}\in P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right): y^m > y_t^m \textit{ implies } y_t^l > y^l \textit{ for } l\neq m.$$

In words, DMU t is Pareto-Koopmans efficient if it is impossible to increase output m without decreasing any other output l for budget z_t, prices $\mathbf{p}_t$ and $\mathbf{P}_t$ under the given production technology (captured by $P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right)$).

So far, we have implicitly assumed that the vector of production functions $f(\mathbf{q}_t^1,\ldots,\mathbf{q}_t^M,\mathbf{Q}_t)$ (and the corresponding set $P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right)$) is known. In practice, however, we do not observe the functions $f^m(\mathbf{q}^m,\mathbf{Q})$. Consistent with the DEA approach that we follow, we avoid using a (typically unverifiable) functional specification for these functions. Rather, we check whether it is possible to construct a production function such that each DMU t satisfies the Pareto-efficiency criterion. Thus, we use the following Pareto-Koopmans output efficiency definition for the data set S:

Definition 2: *The data set S is Pareto-Koopmans output efficient if there exists a vector of production functions* $f(\mathbf{q}_t^1,\ldots,\mathbf{q}_t^M,\mathbf{Q}_t)$ *(and corresponding output producible set* $P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right)$*) such that for each DMU t the output vector* $\mathbf{y}_t$ *is Pareto-Koopmans output efficient*.

We remark that this efficiency criterion is not directly useful, since in principle there are infinitely many possible specifications of $f(\mathbf{q}_t^1,\ldots,\mathbf{q}_t^M,\mathbf{Q}_t)$. Interestingly, however, we can provide an equivalent criterion that has empirical usefulness, as it avoids an explicit construction of $f(\mathbf{q}_t^1,\ldots,\mathbf{q}_t^M,\mathbf{Q}_t)$. In particular, we will derive that the Pareto-Koopmans output efficiency criterion in Definition 2 is met if and only if each DMU *t* produces every output y_t^m (i.e. the *m*th entry of the vector $\mathbf{y}_t$) at a minimal cost when compared to the other DMUs in the data set. In other words, we can equivalently reformulate the above Pareto-Koopmans efficiency condition as a multi-output cost efficiency condition.

To introduce this cost efficiency condition, we will need the following concept of implicit prices for the joint inputs:

Definition 3: *For DMU t, with prices* $\mathbf{P}_t$ *for the joint input, implicit prices* $\mathfrak{P}_t^m \in \mathbb{R}_+^{Njoin}$ *(m = 1,…,M) satisfy* $\sum_{m=1}^{M} \mathfrak{P}_t^m = \mathbf{P}_t$.

These (unobserved) implicit prices represent the fraction of the (observed) aggregate prices of the joint inputs that are borne by the different outputs. This is an intuitive concept from a costing perspective, where some overhead costs are sometimes used by multiple outputs (i.e. they represent joint inputs), but it is unknown to the cost accountant or empirical analyst at which ratio this happens.

Using Definition 3, we can state the following multi-output cost efficiency condition for the data set *S*:

Definition 4: *The data set S is multi-output cost efficient if, for each DMU t, there exist implicit prices such that for each output m the following cost minimization condition holds:*

If, for some DMU s, $y_s^m \geq y_t^m$*, then* $\mathbf{p}_t^{'}\mathbf{q}_t^m + \left(\mathfrak{P}_t^m\right)^{'}\mathbf{Q}_t \leq \mathbf{p}_t^{'}\mathbf{q}_s^m + \left(\mathfrak{P}_t^m\right)^{'}\mathbf{Q}_s$.

Thus, multi-output cost efficiency of the data set *S* requires for each DMU *t* that every output *m* is produced at a minimal cost (when compared to other DMUs *s*), where we use implicit prices to evaluate the joint inputs. In view of our following exposition, it is useful to reformulate the cost minimization condition for each specific output *m* as follows (for DMU *t*):

$$\mathbf{p}_t^{'}\mathbf{q}_t^m + \left(\mathfrak{P}_t^m\right)^{'}\mathbf{Q}_t = \min_{s \mid y_s^m \geq y_t^m} \left(\mathbf{p}_t^{'}\mathbf{q}_s^m + \left(\mathfrak{P}_t^m\right)^{'}\mathbf{Q}_s\right). \tag{1}$$

We can derive the following equivalence between the efficiency concepts in Definitions 2 and 4 (the Appendix contains the proof):

Proposition 1: *The data set S is Pareto-Koopmans output efficient if and only if it is multi-output cost efficient.*

This result states that empirically checking the Pareto-Koopmans efficiency criterion in Definition 2 is equivalent to verifying consistency of each DMU *t* with the cost minimization condition in Definition 4. In the next section, we show that this multi-output cost efficiency condition implies a natural efficiency measure. Subsequently, we show in Section 2.4 that this efficiency measure can be computed through standard linear programming, which makes it easily implementable.

2.3 Efficiency measurement

Suppose we want to evaluate DMU *t* in terms of the multi-output cost efficiency criterion in Definition 4. We start from the cost minimization condition (1) for each specific output *m*. For a given specification of the implicit prices $\mathfrak{P}_t^m$, we can define the minimal cost for output *m* as

$$c_t^m\left(\mathfrak{P}_t^m\right) = \min_{s|y_s^m \geq y_t^m}\left(\mathbf{p}_t^{'}\mathbf{q}_s^m + \left(\mathfrak{P}_t^m\right)^{'}\mathbf{Q}_s\right) \tag{2}$$

so that condition (1) requires $\mathbf{p}_t^{'}\mathbf{q}_t^m + \left(\mathfrak{P}_t^m\right)^{'}\mathbf{Q}_t = c_t^m\left(\mathfrak{P}_t^m\right)$. When considering all outputs *m* together, this naturally suggests the following measure of cost efficiency:

$$CE_t\left(\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^M\right) = \frac{\sum_{m=1}^{M} c_t^m\left(\mathfrak{P}_t^m\right)}{\sum_{m=1}^{M}\mathbf{p}_t^{'}\mathbf{q}_t^m + \mathbf{P}_t^{'}\mathbf{Q}_t}. \tag{3}$$

Clearly, we have $0 \leq CE_t\left(\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^M\right) \leq 1$, with lower values indicating less cost efficiency (or more cost inefficiency). The value of $CE_t\left(\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^M\right)$ has a natural degree interpretation: for given $\mathfrak{P}_t^m$, it captures the extent to which the actual cost $\left(\sum_{m=1}^{M}\mathbf{p}_t^{'}\mathbf{q}_t^m + \mathbf{P}_t^{'}\mathbf{Q}_t\right)$ exceeds the minimal cost $\left(\sum_{m=1}^{M} c_t^m\right)$ for the (multi-dimensional) output that is produced.

However, the cost efficiency measure $CE_t\left(\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^M\right)$ is not directly useful because it requires a prior specification of the implicit prices $\mathfrak{P}_t^m$. In empirical applications, we typically do not observe these prices. In this respect, we recall that Definition 4 (only) requires that there exists *at least one* specification of the implicit prices such that each observation is cost efficient. As such, we can use the following cost efficiency measure in practical efficiency analysis:

$$CE_t = \max_{\mathfrak{P}_t^m \in \mathbb{R}_+^{Njoin} : \sum_m \mathfrak{P}_t^m = \mathrm{P}_t} CE_t\left(\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^M\right). \tag{4}$$

In words, this cost efficiency measure selects those implicit prices that maximize the cost efficiency of DMU *t*. Intuitively, these implicit prices can be interpreted as most favorable prices for evaluating the public inputs. In fact, such most favorable pricing is implicitly used in DEA; see our discussion of LP-2 below.

Similar to before, we have that $0 \leq CE_t \leq 1$, with lower values indicating less cost efficiency; and the degree interpretation of $CE_t\left(\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^M\right)$ carries over to CE_t (but now for the endogenously selected $\mathfrak{P}_t^m$). Clearly, DMU *t* meets the multi-output cost efficiency criterion in Definition 4 if and only if CE_t = 1. The data set *S* is multi-output cost efficient (and thus Pareto-Koopmans output efficient; see Proposition 1) if and only if each DMU *t* is cost efficient (i.e. $CE_t\left(\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^M\right)$ = 1 for all *t*).

Importantly, the multi-output cost efficiency measure CE_t can naturally be decomposed in terms of output-specific cost efficiencies. To see the decomposition, let $\mathfrak{P}_t^{m*}$ solve the max problem in (4), i.e.

$$\left(\mathfrak{P}_t^{1*},\ldots,\mathfrak{P}_t^{M*}\right) = \underset{\mathfrak{P}_t^m \in \mathbb{R}_+^{Njoin}:\sum_m \mathfrak{P}_t^m = \mathrm{P}_t}{\arg\max} CE_t\left(\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^M\right). \tag{5}$$

Correspondingly, we have

$$CE_t = \frac{\sum_{m=1}^{M} c_t^m\left(\mathfrak{P}_t^{m*}\right)}{\sum_{m=1}^{M} \mathbf{p}_t^{'}\mathbf{q}_t^m + \mathbf{P}_t^{'}\mathbf{Q}_t}. \tag{6}$$

Using this, we can write

$$CE_t = \sum_{m=1}^{M} w^m CE_t^m, \tag{7}$$

where

$$w_t^m = \frac{\mathbf{p}_t^{'}\mathbf{q}_t^m + \left(\mathfrak{P}_t^{m*}\right)^{'}\mathbf{Q}_t}{\sum_{m=1}^{M} \mathbf{p}_t^{'}\mathbf{q}_t^m + \mathbf{P}_t^{'}\mathbf{Q}_t} \text{ and } CE_t^m = \frac{c_t^m\left(\mathfrak{P}_t^{m*}\right)}{\mathbf{p}_t^{'}\mathbf{q}_t^m + \left(\mathfrak{P}_t^{m*}\right)^{'}\mathbf{Q}_t}.$$

In this decomposition, CE_t^m measures the cost efficiency of DMU *t* in producing output *m*, while w_t^m represents the weight of this output in the overall (multi-output) cost efficiency measure CE_t. More specifically, the output-specific efficiency measure CE_t^m (always between 0 and 1) expresses how cost efficient DMU *t* is at producing output *m*. Next, the weight w_t^m (also between 0 and 1) represents the share of the total budget that is allocated to output *m* (for the given implicit prices $\mathfrak{P}_t^{m*}$). Ex post, this can be interpreted as the weight allocated to output *m* in the calculation of the multi-output efficiency measure CE_t.

We believe the decomposition in (7) has substantial practical value because the output-specific efficiency measures can guide DMUs when evaluating the cause of their observed inefficiency as well as when planning actions to improve efficiency. In Section 3, we will illustrate the application of the decomposition for managerial purposes.

2.4 Practical implementation

A particularly attractive feature of the measure CE_t in (3) is that it can be computed through linear programming (LP). Actually, the solution of the LP problem also gives the implicit prices $\mathfrak{P}_t^{m*}$ that solve the maximization problem in equation (4). In turn, this enables us to compute the output-specific cost efficiencies CE_t^m and the corresponding weights w_t^m, and so to conduct the decomposition of CE_t in equation (7).

As we will explain, the maximization problem in equation (4) is equivalent to the following LP problem (**LP-1**):

$$\begin{aligned}
&\max_{c_t^m \geq 0, \mathfrak{P}_t^m \in \mathbb{R}_+^{Njoin}} \frac{\sum_{m=1}^{M} c_t^{\,m}}{\sum_{m=1}^{M} \mathbf{p}_t^{'}\mathbf{q}_t^m + \mathbf{P}_t^{'}\mathbf{Q}_t} \\
&s.t. \\
&\text{(C-1)} \ \sum_{m=1}^{M} \mathfrak{P}_t^m = \mathbf{P}_t \\
&\text{(C-2)} \ \forall m : c_t^m \leq \mathbf{p}_t^{'}\mathbf{q}_s^m + \left(\mathfrak{P}_t^m\right)^{'} \mathbf{Q}_s \ \ \forall s : y_s^m \geq y_t^m
\end{aligned}$$

In this problem, the constraints $\mathfrak{P}_t^m \in \mathbb{R}_+^{Njoin}$ and (C-1) make sure that the endogenously selected implicit prices $\mathfrak{P}_t^m$ satisfy Definition 3. Next, for given prices $\mathfrak{P}_t^m$, the constraint (C-2) ensures that c_t^m in LP-1 satisfies equation (3), which defines $c_t^m\left(\mathfrak{P}_t^m\right)$. As a result, we obtain that the solution to LP-1 effectively solves the max problem in equation (4) and vice versa, i.e. the values $\mathfrak{P}_t^{m*}$ defined in equation (5) and the corresponding values $c_t^m\left(\mathfrak{P}_t^{m*}\right)$ solve LP-1.

So far, we have assumed that the input price vectors $\mathbf{p}_t$ and $\mathbf{P}_t$ are exactly observed. However, in many empirical applications such exact price information is not available.[9] Attractively, our cost efficiency measures (including the corresponding LP characterization) can easily be adjusted to account for such incomplete price information. Consistent with usual practice in DEA, we use "most favorable" prices for evaluating the output-specific and joint inputs in the absence of exact price information: we adjust LP-1 so that it selects prices that maximize the efficiency of DMU *t*. In a certain sense, such most favorable prices may be interpreted as *shadow prices* that support cost efficient behavior of the evaluated DMU. Intuitively, (most favorable) shadow prices give each DMU *t* the "benefit of the doubt" in the efficiency evaluation exercise.[10]

More formally, the use of shadow prices $\hat{\mathbf{p}}_t$ and $\hat{\mathbf{P}}_t$ for the inputs obtains the following LP problem (**LP-2**):

$$
\begin{aligned}
&\max_{\substack{c_t^m \geq 0, \mathfrak{P}_t^m \in \mathbb{R}_+^{Njoin}, \\ \hat{\mathbf{P}}_t \in \mathbb{R}_+^{Njoin}, \hat{\mathbf{p}}_t \in \mathbb{R}_+^{Nspec}}} \sum_{m=1}^{M} c_t^m \\
&s.t. \\
&\text{(C-1)}\ \sum_{m=1}^{M} \mathfrak{P}_t^m = \hat{\mathbf{P}}_t \\
&\text{(C-2)}\ \forall m : c_t^m \leq \hat{\mathbf{p}}_t' \mathbf{q}_s^m + \left(\mathfrak{P}_t^m\right)' \mathbf{Q}_s \ \ \forall s : y_s^m \geq y_t^m \\
&\text{(C-3)}\ \sum_{m=1}^{M} \hat{\mathbf{p}}_t' \mathbf{q}_t^m + \hat{\mathbf{P}}_t' \mathbf{Q}_t = 1
\end{aligned}
$$

This LP problem has a readily similar interpretation as LP-1. The only difference is the normalization constraint (C-3) in LP-2. This constraint makes that we can give the objective function of LP-2 a similar ratio interpretation as the objective function of LP-1: because of (C-3) we have $\sum_{m=1}^{M} c_t^m = \left(\sum_{m=1}^{M} c_t^m \,/\, \sum_{m=1}^{M} \hat{\mathbf{p}}_t' \mathbf{q}_t^m + \hat{\mathbf{P}}_t' \mathbf{Q}_t\right)$.[11]

To further clarify the link with more standard DEA analysis, we consider the dual formulation of LP-2. To define the dual problem, we let D_t^m denote the set of DMUs that dominate DMU *t* in output *m*, i.e. $D_t^m = \{s \mid y_s^m \geq y_t^m\}$. Further, let θ_t represent the dual variable associated with constraint (C-3) and λ_s^m the dual variable associated with the constraint (C-2) for each output *m and* DMU *s*. We can then formulate the dual problem as follows (LP-3):

[9] See, for example, Kuosmanen et al. (2006) for a discussion of instances where reliable price information is not readily available. Our application in Section 3 contains another example.

[10] This idea of shadow pricing also underlies the so-called multiplier formulation of standard DEA models (see, for example, Cooper et al. 2000). Cherchye et al. (2007) provide a detailed discussion of the "benefit of the doubt"-interpretation of DEA models in the specific context of composite indicator construction.

[11] In fact, in their original DEA paper Charnes et al. (1978) used a similar normalization constraint to convert their initial fractional programming problem into an equivalent linear programming problem.

$$
\begin{aligned}
&\min_{\theta_t \in \mathbb{R}, \lambda_s^m \geq 0} \theta_t \\
&s.t. \\
&\text{(D-1) } \forall m: \sum_{s \in D_t^m} \lambda_s^m \mathbf{Q}_s \leq \theta_t \mathbf{Q}_t \\
&\text{(D-2) } \forall m: \sum_{s \in D_t^m} \lambda_s^m \mathbf{q}_s^m \leq \theta_t \mathbf{q}_t^m \\
&\text{(D-3) } \forall m: \sum_{s \in D_t^m} \lambda_s^m = 1
\end{aligned}
$$

In this formulation, θ_t measures the efficiency of DMU *t* as a proportional reduction of the inputs; the specificity of our efficiency measurement model is that it simultaneously accounts for joint inputs (see constraint (D-1)) and output-specific inputs (see constraint (D-2)). Similar to standard DEA models, the benchmark input vectors are constructed as (convex) combinations of existing DMUs, with every variable λ_s^m representing the weight of each DMU *s*. In common DEA terminology, the variables λ_s^m are referred to as intensity variables. We note that problem LP-3 obtains separate benchmark (joint and output-specific) input vectors ($\sum_{s \in D_t^m} \lambda_s^m \mathbf{Q}_s$ and $\sum_{s \in D_t^m} \lambda_s^m \mathbf{q}_s^m$) for every different output *m*. This feature relates to the particular nature of our approach, which explicitly recognizes that each different output is characterized by its own production technology (and, therefore, its own benchmark input).

One final remark concerns the shadow price problem LP-2 and its dual formulation LP-3. The corresponding efficiency analysis can be strengthened by imposing price information in the form of additional constraints that define a feasible range for the relative prices; for example, such shadow price constraints may rule out the extreme cases where the relative price of a commodity approaches zero or infinity. The technical questions related to incorporating such shadow price restrictions have been discussed extensively in a DEA context, most commonly under the label 'weight restrictions' or 'assurance regions' (see, for example, Allen et al.(1997) and Pedraja-Chaparro et al.(1997), for surveys, and Kuosmanen et al. (2006) for more recent developments). [12] These tools are readily adapted to the current set-up. Typically, DEA shadow price restrictions are linear and, as such, they do not interfere with the linear nature of LP-2. As a specific illustration, we will use price restrictions in our application.

[12] See, for example, Podinovski (2004) for a discussion on incorporating weight restrictions in (dual) DEA problems such as LP-3.

3. Application

3.1 Data

Our empirical application uses data from a large service company active in a European country. It delivers its services to the end customer through 290 offices (i.e. DMUs) that are spread among the country. The offices only differ from each other in terms of their size, which is linked to the size and the population density of the geographical area they operate in. Further, all 290 offices can deliver the same 7 standardized outputs to the end customer, with the corresponding output targets exogenously given (i.e. DMU managers do not have control over the output quantities). As a result, the goal of each office is cost minimization for a given output, which complies with the cost-oriented approach of our methodology.

The company under investigation has its own ABC system, which is implemented at the office level. This implies that we have information about the inputs, resource drivers, costs of activities, activity drivers and outputs for each DMU. In consultation with the company management, we aggregated the variables in the original ABC model, resulting in a model with 7 inputs (i.e. cost categories), 7 activities and 7 outputs. Each DMU uses three types of inputs: labor, transport, and other overhead costs. More specifically, the model contains 3 categories of labor, 3 categories of transport, and 1 category of other overhead costs, which yields a total of 7 inputs. The labor and transport subcategories differ from each other in terms of their relationship with the activities. We treat them as distinct inputs because pooling heterogeneous cost categories can decrease the accuracy of the costing system (Labro and Vanhoucke 2007). Labor categories consist of the wages paid to different types of employees. Transport expenditures are fuel costs, maintenance costs and depreciation for different types of vehicles. Other overhead costs consist of all other expenditures made at the DMU level such as pay of the DMU manager, maintenance of the building,… For each DMU, we obtained expenditure data for every input. Specifically, we treat expenditures as aggregate input quantity indices (i.e. quantities multiplied by prices, with price differences correcting quality differences in the quantity composition). Due to confidentiality and strict Non-Disclosure Agreements, we cannot provide details on the activities, which cover the entire production process of the DMUs, and outputs of the ABC system.

Panel A of Table 1 provides descriptive statistics for the 7 inputs. The large difference between the minimum and maximum values of the different inputs reflects a large variation across the 290 DMUs. We should also mention that some DMUs do not use some of the inputs 4, 5, 6 and 7. Based on the mean relative weights, we can conclude that inputs 1, 2 and 7 are most important. Panel B of Table 1 gives summary statistics for the activities. Activities 3 and 4 are the most input consuming activities. Panel C of Table 1 shows the summary statistics for the outputs. Output 1 is the most important output and takes an average share of 90,78%. The other outputs seem to be far less important. At this point, however, we note that it would be misleading to only consider this output in our efficiency analysis, as it is shrinking in volume year after year and the management is explicitly focusing its attention towards the other outputs.

- insert Table 1about here-

We believe that this empirical application is well suited for demonstrating the practical usefulness of our newly developed efficiency measurement methodology: ABC data are available at the office level, all offices work in a standardized way, which makes them comparable (i.e. DMUs operate in a similar technological environment), offices are quite heterogeneous in terms of inputs used and outputs produced, and cost efficiency (i.e. cost minimization for given output targets) is an appropriate efficiency concept.

3.2 Efficiency results

Our empirical exercise considers four different efficiency measurement models: the first three models involve different specifications of the outputs-specific and joint inputs for calculating the (multi-output) cost efficiency measure presented in Section 2 (CE_t); the fourth model uses a standard cost efficiency measure (SCE_t, which we define below) and will be used as a benchmark model. In each model we use shadow prices to evaluate the different inputs. In doing so, we employ shadow price restrictions to exclude unrealistic input prices; these restrictions have been specified in consultation with the company management.

Our first three models solve the problem LP-2 (complemented with linear shadow price restrictions) to compute CE_t for each DMU *t* under different selections of the output-specific and joint inputs. The first model (*BASIC*) is our core model and considers 6 output-specific inputs (input 1-6) and 1 joint input (input 7). We classified input 7 as a joint input as this input is a facility-level input (see Section 2.1). The company management is aware that the allocation of this input is somewhat ‘artifical’ and agrees with the way in which we distinguish between output-specific inputs and joint inputs. The interpretation of this model is that the ABC system allows us to allocate 6 inputs directly to the outputs, while one input cannot be allocated to any specific output (and, thus, is ‘shared’ by the different outputs). In the second model (*ALL_ALLOCATED*), we use the original ABC system in which input 7 is also allocated to the outputs. By contrast, in our third model (*NONE_ALLOCATED*) we do not use any information provided by the ABC system and, thus, all inputs are treated as joint inputs. This model broadly coincides with the model of Cherchye et al. (2008).

We believe that it is useful to compare our findings for these three models with a 'standard' cost efficiency measurement model, which does not consider jointly used inputs and/or inputs allocated to specific outputs. This benchmark model (*BENCHMARK*) is defined as follows for each DMU t:[13]

$$SCE_t = \left[\frac{c_t}{\sum_{m=1}^{M} \mathbf{p}_t^{'} \mathbf{q}_t^{m} + \mathbf{P}_t^{'} \mathbf{Q}_t} \middle| c_t = \min_{s|\mathbf{y}_s \geq \mathbf{y}_t} \sum_{m=1}^{M} \mathbf{p}_t^{'} \mathbf{q}_s^{m} + \mathbf{P}_t^{'} \mathbf{Q}_s \right] \tag{8}$$

Thus, for given prices $\mathbf{p}_t$ and P_t this measure divides the minimal cost c_t by the actual cost $\sum_{m=1}^{M} \mathbf{p}_t^{'} \mathbf{q}_t^{m} + \mathrm{P}_t^{'} \mathbf{Q}_t$; the minimal cost is defined over all DMUs s that produce at least the same amount as DMU t of all different outputs (i.e. $\mathbf{y}_s \geq \mathbf{y}_t$). The essential difference between the measure SCE_t and our multi-output cost efficiency measure CE_t is that this last measure accounts for (interdependent) output-specific production technologies; this complies with the fact that CE_t is composed of output-specific cost efficiency measures CE_t^m in (7). In turn, this implies that the newly proposed measure CE_t generally has more discriminatory power than the standard measure SCE_t (because CE_t incorporates more prior information about the underlying production process). We will illustrate this last point in our empirical results.

For each of the four models, we consider two efficiency assessment exercises. In Section 3.2.1, we present the efficiency results without controlling for exogenous variables that may have an impact on DMU efficiency and without correcting for possible outlier behavior of particular DMUs. Subsequently, Section 3.2.2 reports on a second exercise, in which we control for population density as a relevant exogenous variable and simultaneously account for the possibility of outlier behavior. Our selection of population density as the (sole) exogenous variable that is controlled for is the result of consultation with the company management. Next, explicitly accounting for outlier behavior should obtain efficiency results that are more robust (e.g. with respect to measurement errors for inputs and outputs, and non-comparability of DMUs due to (unobserved) heterogeneity of the production environment). In this second exercise, we make use of a probabilistic method that has recently been proposed in a DEA context and that is extensively discussed by Daraio and Simar

[13] For simplicity, we define the standard cost efficiency measure without shadow prices. Including shadow prices proceeds analogously as before. See Cherchye and Vanden Abeele (2005) for a more detailed discussion of this cost efficiency measure. These authors also provide a linear programming formulation to compute the measure when using shadow prices for evaluating the inputs. As indicated above, we will use this shadow price formulation in our empirical exercise, in which we will include the same shadow price restrictions as for the other three efficiency measurement models.

(2007).[14] This also shows that our new DEA-based methodology can be easily combined with this probabilistic method (as well as with other existing DEA methodologies).

3.2.1. Without control for exogenous variables or outlier behavior

Panel A of Table 2 reports the results for the four efficiency models without control for any exogenous variable and without correction for outlier behavior in the data. Considering the results for the *BASIC*-model, we find that only 10% of the DMUs are efficient and that the average cost reduction potential amounts to 20%. This last result implies that the average office can produce the same output with 20% fewer costs. The results for the *BASIC*-model also show that this model has considerable discriminatory power. This is an interesting property of our methodology, especially when taking into account the attractive structure of the model (with reasonable behavioral assumptions and minimal (unverifiable) production assumptions; see Section 2). In economic terms, our results suggest that, at the aggregate company level, the same output can be produced after reducing totals costs with 123.243.958 EUR. As yet another point of reference, such a cost decrease would imply an increase of the company's EBIT (i.e. earnings before interest and tax) of as much as 33%, ceteris paribus.

The results for the *ALL_ALLOCATED*-model, which are qualitatively similar to the results for the *BASIC*-model, show that 14% of the DMUs are efficient and the average DMU can produce the same output with a cost reduction of 11%. The small differences between the results for the *ALL_ALLOCATED*- and *BASIC*-models should not be too surprising as the only difference between the two models lies in the treatment of input 7, which accounts for only one sixth of the total costs (i.e. input 7 is considered as a joint input in the *BASIC*-model and an output-specific input in the *ALL_ALLOCATED*-model). However, the differences between both models are substantial enough to make clear that the classification of an input as joint or output-specific matters for the efficiency analysis and for the conclusions that are drawn from it. As for this particular application, we prefer to focus on the *BASIC*-model (and, thus, to treat input 7 as a joint input) as this model better reflects the particular environment of the company.

Next, the empirical results for the *NONE_ALLOCATED*-model are consistent with our expectations. As this model puts very little prior structure by treating all inputs as joint inputs (i.e. no input is specifically allocated to the outputs), we may reasonably expect that the model will have low explanatory power. The results show that almost 90% of the DMUs is declared efficient and the average cost reduction potential is only 2%.

Finally, we consider the results of the *BENCHMARK*-model, which uses the standard cost efficiency measure defined in (8). We find that this model has very low discriminatory power for the given data set: almost all DMUs are efficient. Comparing these findings with our results for the

[14] The original ideas of this method were presented in Cazals et al. (2002) and Daraio and Simar (2005).

BASIC- and *ALL_ALLOCATED*-models provides a strong empirical argument pro using our newly proposed method: the explicit distinction between output-specific and joint inputs in the efficiency assessment does substantially contribute to the discriminatory power of the analysis. In turn, this also pleads for using detailed cost accounting data (generated by an ABC system), which effectively enables such a distinction.

3.2.2. With control for population density and outlier behavior

As mentioned earlier, we also computed efficiency results for the same four models when treating population density as an exogenous variable impacting DMU efficiency, and while accounting for outlier behavior. To this end, we combined our method with the probabilistic order-alpha method of Daouia and Simar (2007). We refer to Daouia and Simar (2007) for a detailed treatment of the method, and restrict to sketching the main idea. The probabilistic method starts by estimating a nonparametric kernel density function through the values of the exogeneous variable Z (in our case population density), using a bandwidth h that is determined by cross-validation techniques. Then, it restricts the set of potential comparison partners for each DMU t (with value Z_t for the exogenous variable) to those DMUs of which the corresponding Z value lies within the range $[Z_t - h, Z_t + h]$; as a result, DMU t will only be compared to other DMUs that have a Z value close to Z_t.[15] Specifically, the method repeatedly draws random subsamples (with replacement) from this restricted set of potential comparison partners. For each draw it computes DMU t's cost efficiency, defining a subsample-specific efficiency value. The outlier-robust efficiency measure is then calculated as the average (over all draws) subsample-specific efficiency values. The following efficiency results pertain to this robust measure (for all four efficiency measurement models under consideration).[16]

Panel B of Table 2 summarizes our findings. A first observation is that the average efficiency value and the number of efficient DMUs for the *BASIC*-model are substantially higher than the corresponding values in Panel A of the same table (i.e. without control for population density and outlier behavior). This suggests that differences in population density as well as outlier behavior may have an important influence on the efficiency results. However, even if we control for these factors, our *BASIC*- model still has a lot of discriminatory power. Specifically, 66,55% of the DMUs are identified as cost inefficient and the mean cost reduction potential still amounts to 6%. The economic impact of this result is still significant: at the aggregate company level, a potential cost reduction of

[15] To be precise, to compute CE_t, the original set of comparison partners for each output m (with corresponding CE_t^m in (7)) is the set of DMUs s such that $y_s^m \geq y_t^m$. Similarly, the original set of comparison partners to compute SCE_t is the sets of DMUs s such that $\mathbf{y}_s \geq \mathbf{y}_t$ (see (8)). The restricted sets of comparison partners contain those DMUs s that additionally satisfy the requirement that Z_s lies within $[Z_t - h, Z_t + h]$.

[16] In our exercise, we conducted 200 random draws for calculating these robust measures. In each draw, the number of observations in the subsample equaled 80% of the number of observations in the restricted set of potential comparison partners (where we round to the first higher integer if necessary).

36.973.187 EUR could be realized without decreasing the output level. Such a cost reduction would increase the EBIT with 10%, ceteris paribus.

Next, we find that the discriminatory power also decreased for the *ALL_ALLOCATED*- and *NONE_ALLOCATED*-models compared to the same models without control for population density and outlier behavior. Generally, the results for these models yields the same qualitative conclusions as before. First, we observe some differences between the results for the *ALL_ALLOCATED*- and *BASIC*- models, which indicates that treating input 7 as a joint input matters for the analysis. Second, we observe that the discriminatory power of the *NONE_ALLOCATED*-model is very low, which again shows that using information about the allocation of the output-specific inputs may substantially enhance the efficiency analysis.

Finally, the *BENCHMARK*-model with a control for population density loses all discriminatory power. If one were to use this method only, it would seem as all offices are operating efficiently. Once again, this result provides a strong empirical argument for using our newly developed efficiency measurement method.

3.3 Managerial Implications

Companies often have multiple business units or offices (i.e. DMUs) that produce identical outputs. A major task of top management is to monitor the efficiency of these DMUs in converting inputs into outputs, and to take appropriate decisions based on the efficiency assessment. Examples of such decisions are evaluation of business unit managers and the linked bonus payments, the installment of benchmarking programs and initiation of improvement actions for bad performing business units, and potentially the dismissal of business unit managers or closing of bad performing business units.

However, accurate efficiency assessment is a complex task for several reasons. First, production processes with multiple outputs are typically characterized by inputs that can be directly allocated to the specific outputs (output-specific inputs) as well as inputs that simultaneously assist in the production of different outputs (joint inputs). The labor cost of employees of a department of a typical supermarket store, for instance, can be directly attributed to the products of that department. The salary of the store manager, however, cannot be attributed to a product or product group. The existence of joint inputs thus necessitates the use of a method that allocates the joint inputs to the multiple outputs in a way that does not bias the efficiency assessment at the disadvantage of the business unit. Second, business units do not produce the same output mix. Third, even standardized business units operate in different environments. They are subject to different environmental (i.e., exogenous) factors that are beyond their control but influence their efficiency (e.g. population density, average household income,...). As business units should only be held accountable for their inefficiency resulting from controllable factors, and not for the influence of the environment they operate in, a refined methodology is necessary.

We believe that our newly developed methodology has some unique benefits that can improve efficiency assessments of business units and, as a consequence, firm performance. A first benefit is that including information about the allocation of the output-specific inputs to the different outputs substantially improves the discriminatory power of the efficiency assessment. Evidently, an efficiency measurement methodology with more discriminatory power has a greater managerial relevance, as DMU-managers can only be motivated to initiate improvement actions if their DMU is identified as inefficient by the efficiency assessment. Furthermore, by treating some inputs as joint inputs and by allocating these inputs to the outputs in a way that does not harm the efficiency result of the particular business unit, our methodology calculates efficiency in a conservative way and takes into account the particular features of the production process. Finally, our methodology can be easily combined with well-known extensions of DEA-based efficiency assessments (e.g. to control for exogenous factors and outlier behavior) so that the benefits of these extensions also pertain to our methodology. Taken together, assessing the efficiency of DMUs by means of our methodology will make the results of the efficiency assessment more acceptable for business unit managers, lead to more improvement actions and, consequently, higher realized cost reductions and improved firm performance.

A second interesting feature of our methodology is that it allows us to decompose the overall efficiency value of a DMU in output-specific efficiency values and corresponding weights (revealing the importance of each individual output in the overall efficiency value; see our discussion of (7) in Section 2). Such a decomposition can lead to more focused improvement actions compared to approaches that do not decompose the overall efficiency value. Indeed, without a decomposition of the overall efficiency value, managers of multi-output DMUs have no clear guidance in terms of the outputs on which they should focus in order to correct the inefficiency that is detected. Taken together, the main distinguishing features of our methodology pertain to the identification of inefficient DMUs and to the fact that it provides managers with more guidance for the installment of improvement actions.

To show the practical usefulness of the decomposition of the overall efficiency value of a DMU, we provide a specific example taken from our application. Panel C of Table 2 reports the output-specific efficiencies and the output weights for three DMUs (A, B and C) that attain the same overall efficiency score (i.e. 0,65). The level of the overall efficiency value indicates that each DMU can produce the same combination of outputs with a cost level that is 35% below the current cost level. While standard methods for efficiency assessment, which typically do not decompose the overall efficiency measure, would stop here, our methodology allows us to go further by analyzing the sources of this cost inefficiencies at the individual output level.

Careful inspection of the output-specific efficiencies reveals some notable differences across the three DMUs. Output 1, for instance, is produced efficiently in DMU B, while DMUs A and C turn out to be inefficient in the production of this output. Considering the weights for output 1 shows that this output is more important for DMU A (i.e. a weight of 0,51) than for DMU B and C (i.e. a weight of respectively 0,11 and 0,06). Summarizing, this example shows that output-specific efficiencies and the corresponding weights can vary a lot between DMUs, which emphasizes the importance of providing this information to managers in order to help them to increase the efficiency of their DMUs.

When considering the other outputs of the DMUs in more detail, we find that the focus of the improvement actions may substantially vary across DMUs. For example, DMU B is performing quite well for outputs 4 and 5 (with output 4 much more important than output 5). By contrast, its cost efficiency is much lower for output 3, which is almost as important as output 4. However, the most problematic is output 6, which is only slightly less important than output 3, but has dramatically low efficiency. We also note that the efficiency of outputs 2 and 7 is low, but these outputs are only marginally important for the cost efficiency of DMU B. Taken together, our advice for DMU B is to focus mainly on the production of output 6 and, to a somewhat lesser extent, output 3.

A similar analysis for DMUs A and C yields the following conclusions. First, DMU A can improve its overall efficiency by focusing on output 1, which is very important and is characterized by a potential cost reduction of 14%. In addition, this DMU can fruitfully focus on a more efficient production of outputs 2 and 3, which are a bit less important but characterized by much more room for improvement than output 1. Finally, DMU C should in particular concentrate on output 6, which is both highly important and produced quite inefficiently.

- insert Table 2 about here -

4. Conclusion

Companies often have multiple business units in which the same outputs are produced. Well-known examples of such companies are Wal-Mart, Home Depot and Mc Donald's. An assessment of the efficiency of the different business units is necessary to manage such companies in an adequate way. This study develops a new DEA-based methodology that improves the efficiency measurement of multi-output DMUs and provides guidance for the improvement actions to restore inefficiency. The distinguishing feature of our methodology is that we include information about the decomposition of the inputs to the outputs. Interestingly, companies often have such information available in their ABC systems.

This new approach to efficiency measurement enriches the production efficiency analysis in two different ways. First, including information about the input decomposition substantially improves the discriminatory power of the efficiency assessment. Specifically, our new methodology is better able to detect productive inefficiencies, which should lead to more improvement actions and higher realized cost savings. A second interesting contribution of our method is that it allows for decomposing the overall efficiency in output-specific efficiencies. Overall cost efficiency measures indicate how well a particular DMU performs in the aggregate, but it does not generate any direct guidance as to which actions can effectively improve the observed inefficiencies. By contrast, output-specific efficiency measures effectively identify the outputs on which DMUs should focus to remedy the observed inefficiency. Given that business units typically have limited resources to remedy inefficiencies, our methodology helps to better allocate these scarce resources to the outputs that contribute the most to the inefficiency that is observed. Summarizing, our methodology will lead to more improvement actions as well as more focused improvement actions.

This study also contributes to the literature on costing systems: our empirical application shows that our methodology is naturally complementary to ABC systems, and that ABC information can be particularly useful for assessing the efficiency of business units. We believe that this potential of using ABC data for efficiency assessment can be an important decision criterion to invest in such costing systems.

We see multiple avenues for follow-up research. First, as for empirical applications, we have suggested using ABC data to obtain information about the decomposition of the output-specific inputs to the different outputs. Although ABC is a more accurate costing method than volume-based costing methods, it is unlikely to be error-free. Furthermore, previous research has shown that the accuracy of ABC systems depends the characteristics of the economic environment, such as diversity in the resource consumption patterns (Labro and Vanhoucke 2007). Future research could investigate how the determinants of the accuracy of costing systems influence the accuracy of the efficiency assessments.

Next, at a methodological level, our approach allows for a richer type of efficiency analysis, because it explicitly recognizes that different outputs are characterized by own (possibly interdependent) production technologies. In this respect, the current study has focused on Pareto-Koopmans efficient output production, because this is the most popular efficiency criterion in the existing literature. However, one may also assume a Nash equilibrium allocation for multi-output production (which need not necessarily be Pareto-Koopmans efficient). Here, one may fruitfully build on Cherchye et al. (2011), who considered this Nash equilibrium criterion in a formally close consumption setting. More generally, we believe that our modeling of output-specific production technologies opens the way for a whole new spectrum of applications of multi-output efficiency analysis.

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Appendix: Proof of Proposition 1

As a first step, we use a standard result in welfare economics, namely: under convex utility possibility sets, any Pareto-efficient allocation can be characterized as a stationary point of a linear social welfare function (see, for example, Mas-Colell, Whinston and Green (1995)). This result is readily translated towards the current setting, which is characterized by convex output producible sets (instead of utility possibility sets). Specifically, we obtain the following equivalence (**Result 1)**:

For a given vector of production functions $f(\mathbf{q}^1,\ldots,\mathbf{q}^M,\mathbf{Q})$*, with producible output set* $P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right)$*:*

$\mathbf{y}_t$ *is Pareto-Koopmans output efficient*

$$\Leftrightarrow$$

there exists $\mu_t \in \mathbb{R}_+^M : \mu_t^{'}\mathbf{y}_t \geq \mu_t^{'}\mathbf{y} \text{ for all } \mathbf{y} \in P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right)$.

As a second step , we can use the following equivalence (**Result 2**):

For a data set S:

there exists $f(\mathbf{q}^1,\ldots,\mathbf{q}^M,\mathbf{Q})$*, with producible output set* $P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right)$,

such that for each DMU t there exists $\mu_t \in \mathbb{R}_+^M$:

$$\mu_t^{'}\mathbf{y}_t \geq \mu_t^{'}\mathbf{y} \text{ for all } \mathbf{y} \in P\left(z_t,\mathbf{p}_t,\mathbf{P}_t\right).$$

$$\Leftrightarrow$$

for each DMU t, there exist implicit prices
such that for each output m: if for some DMU s

$y_s^m \geq y_t^m$*, then* $\mathbf{p}_t^{'}\mathbf{q}_t^m + \left(\mathfrak{P}_t^m\right)^{'}\mathbf{Q}_t \leq \mathbf{p}_t^{'}\mathbf{q}_s^m + \left(\mathfrak{P}_t^m\right)^{'}\mathbf{Q}_s$.

The proof of this second equivalence is directly analogous to the one of Proposition 1 in Cherchye, De Rock and Vermeulen (2008). For compactness, we do not repeat it here. See also Chiappori (1988) and Cherchye, De Rock and Vermeulen (2007, 2011) for formally similar results in a consumption context.

Combining Results 1 and 2, and using Definitions 2 and 4, we get the wanted result:

The data set S is Pareto-Koopmans output efficient

if only if it is multi-output cost efficient.

FIGURE 1: ABC model

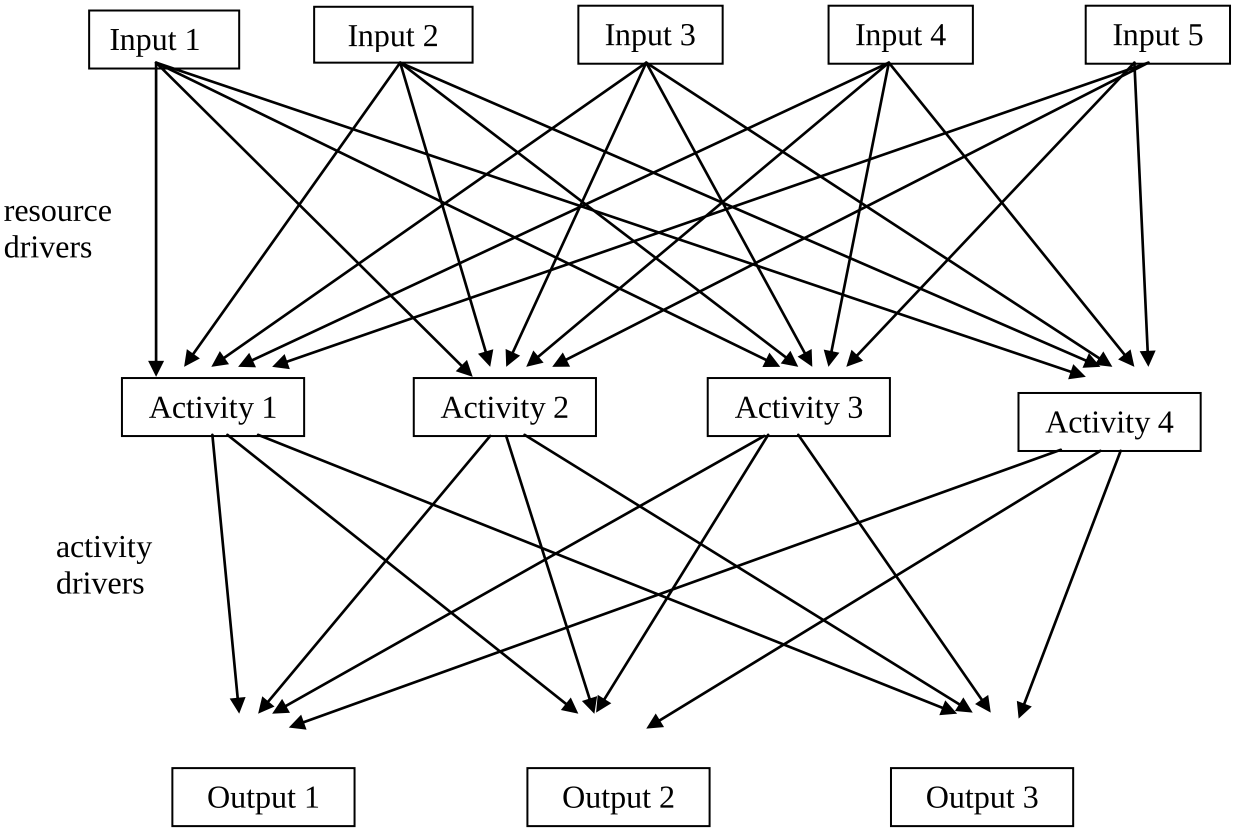

TABLE 1: Summary statistics for input, activities, and outputs

PANEL A: SUMMARY STATISTICS FOR INPUTS

	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6	Input 7
Minimum	2.300,53	6.522,28	2.402,56	0,00	0,00	0,00	0,00
1st Quartile	416.512,75	178.020,86	27.060,68	16.722,70	861,31	99,13	8.224,88
Median	681.166,76	346.869,95	50.897,87	38.132,60	3.913,21	965,87	18.776,13
3rd Quartile	1.235.242,32	641.469,51	96.741,76	69.255,21	10.609,42	5.327,23	84.246,26
Maximum	7.852.652,61	3.106.230,5	807.476,34	512.198,29	50.478,14	555.005,36	5.836.885,8
Mean	1.083.253,08	511.329,46	88.798,65	57.454,91	7.351,60	29.023,07	347.685,06
Stddev	1.256.051,39	493.970,15	113.984,00	68.758,95	9.163,86	76.749,02	873.538,76
Mean Relative Weight	0,51	0,24	0,04	0,03	0,00	0,01	0,16

PANEL B: SUMMARY STATISTICS FOR ACTIVITIES

	Activity 1	Activity 2	Activity 3	Activity 4	Activity 5	Activity 6	Activity 7
Minimum	50.715,01	16.633,58	109.033,49	83.422,97	57.595,20	660,08	28.828,71
1st Quartile	134.417,95	44.086,60	272.126,91	221.109,00	152.653,60	1.749,52	76.409,25
Median	208.236,47	68.297,70	432.590,15	342.535,79	236.486,62	2.710,31	118.371,04
3rd Quartile	365.520,84	119.884,06	763.735,28	601.258,59	415.108,78	4.757,45	207.778,59
Maximum	1.646.721,96	540.094,28	3.239.597,01	2.708.753,15	1.870.122,49	21.432,96	936.071,30
Mean	314.939,21	103.294,23	647.817,50	518.055,03	357.665,06	4.099,10	179.025,71
Stddev	298.032,42	97.749,11	601.898,45	490.244,43	338.464,63	3.879,05	169.415,12
Mean Relative Weight	0,15	0,05	0,30	0,24	0,17	0,00	0,08

PANEL C: SUMMARY STATISTICS FOR OUTPUTS

	Output 1	Output 2	Output 3	Output 4	Output 5	Output 6	Output 7
Minimum	5.647,86	0,00	0,00	11,97	2,06	33,25	2,68
1st Quartile	17.351,95	0,00	65,75	67,88	7,20	117,18	140,65
Median	28.991,22	0,00	1.501,44	99,32	13,69	206,15	240,37
3rd Quartile	48.648,71	0,00	3.240,62	159,19	23,09	369,07	458,22
Maximum	167.844,62	70,00	28.251,79	837,27	78,42	5.515,25	4.171,21
Mean	37.935,31	0,93	2.859,37	134,84	18,22	382,28	456,77
Stddev	30.129,41	5,28	4.540,18	108,95	14,95	574,87	644,00
Mean Relative Weight	0,91	0,00	0,07	0,00	0,00	0,01	0,01

TABLE 2: Efficiency results

PANEL A: EFFICIENCY RESULTS WITHOUT CONTROL FOR POPULATION DENSITY AND OUTLIER BEHAVIOR

Efficiency measure	BASIC	ALL_ALLOCATED	NONE_ALLOCATED	BENCHMARK
Minimum	0,23	0,30	0,52	0,61
1st Quartile	0,68	0,86	1,00	1,00
Median	0,83	0,92	1,00	1,00
3rd Quartile	0,96	0,97	1,00	1,00
Maximum	1,00	1,00	1,00	1,00
Mean	0,80	0,89	0,98	1,00
Stdev	0,18	0,12	0,07	0,03
Efficient DMUs				
Number	29	40	259	285
Percentage	10,00%	13,79%	89,31%	98,28%

PANEL B: EFFICIENCY RESULTS WITH CONTROL FOR POPULATION DENSITY AND OUTLIER BEHAVIOR

Efficiency measure	BASIC	ALL_ALLOCATED	NONE_ALLOCATED	BENCHMARK	POP.DENSITY
Minimum	0,43	0,42	0,59	1,00	5,2
1st Quartile	0,92	0,88	1,00	1,00	16,5
Median	0,98	0,93	1,00	1,00	22,2
3rd Quartile	1,00	0,97	1,00	1,00	34,2
Maximum	1,00	1,00	1,00	1,00	253,5
Mean	0,94	0,91	0,99	1,00	33,9
Stdev	0,10	0,10	0,05	0	38,9
Efficient DMUs					
Number	97	47	270	290	-
Percentage	33,45%	16,21%	93,10%	100%	-

PANEL C: DECOMPOSITION OF OVERALL EFFICIENCY FOR THREE DMUS (weights between brackets)

	Overall	Output1	Output2	Output3	Output4	Output5	Output6	Output7
A	0,65	0,86	0,25	0,56	0,87	0,22	0,00	0,10
		(0,51)	(0,16)	(0,17)	(0,07)	(0,06)	(0,02)	(0,01)
B	0,65	1,00	0,21	0,69	0,94	0,99	0,01	0,01
		(0,11)	(0,02)	(0,23)	(0,29)	(0,10)	(0,21)	(0,04)
C	0,65	0,89	0,40	0,50	0,53	0,84	0,67	1
		(0,06)	(0,05)	(0,09)	(0,09)	(0,04)	(0,67)	(0,00)